AI-1545 M.A./M.Sc. (Final) Mathematics

Term End Examination, 2020-21 Compulsory/Optional Group-Paper-

INTEGRATION THEORY & FUNCTIONAL ANALYSIS

Time: - Three Hours]

[Maximum Marks:100

[Minimum Passing Marks:036

Note: Answer any five questions. All questions carry equal marks.

- 1. State and prove Hahn decomposition theorem.
- 2. State and prove Riesz Representation theorem.
- 3. Let P be a real number such that is $1 \le P < \infty$ and l_p^n denote the linear space of all n-tuples of scalars with the norm of a vector $x = (x_1, x_2, \dots, x_n)$ defined by $||x||_p = (\sum |xi|^p)^{1/p}$ Show that l_p^n is a Banach space.
- 4. Let T be a linear transformation of a normed linear space N into another normed Linear space N'. Then the following statements are equivalent-
 - (i) T is continuous
 - (ii) T is continuous at the origin in the sense that $x_n \to 0 \iff T(x_n) \to 0$
 - (iii) There exists a real number $k \ge 0$ such that $||T(x)|| \le k||x||, \forall x \in N$.
- 5. State and prove closed graph theorem.
- 6. State and prove uniform boundedness theorem.
- 7. (a) Prove that a Banach space is Hilbert space if and only if the Parallelogram law hold.
 - (b) State and prove Bessel's Inequality.
- 8. Let Y be a fixed vector in a Hilbert space it and let f_y be a scalar valued function on H defined $f_y(x) = (x, y), \forall x \in H$
- 9. Let T be on operator on a Hilbert space H then there exist a unique operator T^* on H such that $(Tx,y)=(x,T^*y) \forall x,y \in H$
- 10. (a) If T is an operator an a Hilbert space H, then (Tx,x)=0 for all x in H⇔T=0
 (b) If T is a positive operator an a Hilbert space H. Then I+T is non-singular.